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Compatible volume and taper equation systems for roble and hualo second-growth forests in Chile

Sistema de ecuaciones compatibles de volumen y ahusamiento para renovales de roble y hualo en Chile

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ABSTRACT

Estimates of merchantable volume in second-growth forests are made in an environment of high uncertainty. Therefore, there is a need to develop more flexible predictive models for estimating volume at multi-product level. This research developed compatible volume and taper equation systems for the native tree species roble (*Nothofagus obliqua*) and hualo (*Nothofagus glauca*). Data were obtained from a destructive sampling of 144 individuals of roble and 196 individuals of hualo. For each tree species, 70 % of the sample trees were randomly selected for parameter estimates, and the remaining 30 % for model validation. Two compatible volume and taper equation systems were derived from a segmented and a high-degree polynomial taper model. Error autocorrelation was accounted incorporating a second-order continuous autoregressive error structure CAR(2). A comparison between the two compatible systems was made based on the stem diameter and total tree volume predictive capabilities. The inclusion of CAR(2) in the model error structure allowed to reduce the effect of the autocorrelation generated by the diameter measurements taken along the stem. For both tree species, the segmented taper model presented the best performance for stem diameter predictions in comparison to the polynomial taper model. However, both compatible systems showed similar predictive capabilities for total volume. Based on the results obtained we recommend the use of the compatible volume and taper equation system derived from the segmented taper model.

Keywords: stem diameter, stem taper, error autocorrelation, merchantable volume, *Nothofagus*.

RESUMEN

Las estimaciones de volúmenes comerciales en renovales se realizan en un ambiente de alta incertidumbre. Por lo tanto, existe la necesidad de desarrollar modelos de volúmenes más flexibles que permitan la estimación volumétrica a nivel de productos. Este estudio presenta la construcción de sistemas de ecuaciones compatibles de volumen y ahusamiento para árboles de especies nativas de roble (*Nothofagus obliqua*) y hualo (*Nothofagus glauca*). Los datos se obtuvieron de una muestra destructiva de 144 individuos de roble y 196 individuos de hualo. Para cada especie, un 70 % de los árboles fueron seleccionados aleatoriamente para la estimación de parámetros y el 30 % restante para validación. Se evaluaron dos sistemas compatibles que se derivaron de un modelo fustal segmentado y de un modelo polinómico de alto grado. La autocorrelación de errores fue modelada incorporando una estructura de error autorregresiva continua de segundo-orden CAR(2). La comparación entre ambos sistemas compatibles consideró la capacidad para predecir diámetros fustales y volumen total. La incorporación de CAR(2) permitió reducir el efecto de la autocorrelación producida por la medición de diámetros a lo largo del fuste. Para ambas especies, el modelo segmentado presentó el mejor desempeño respecto a la predicción de diámetros en comparación al modelo fustal polinómico. Sin embargo, ambos sistemas presentaron una capacidad predictiva similar para el volumen total. Finalmente, se recomienda el uso del sistema de volumen compatible derivado del modelo fustal segmentado.

Palabras clave: diámetro fustal, ahusamiento fustal, autocorrelación de errores, volumen comercial, *Nothofagus*.

INTRODUCTION

A sustainable management of second-growth forests in Chile requires to increase the quantity and quality of wood produced. It is estimated that this type of forests structure covers a total area of 4.4 million hectares with potential for forest management. Moreover, they have the highest growth rates and contain the tree species with the highest commercial value that belong mainly to the genus *Nothofagus* (Ulloa, 1984). In the Chilean central region, the Roble-Hualo forest type (after the common name of its two major tree species, i.e. *Nothofagus obliqua* (Mirb.) Oerst and *Nothofagus glauca* (Phil.) Krasser) covers an area of 205,923 ha of which 175,701 ha (85%) are classified as second-growth forests (CONAF, 2021). Past studies have shown that these native forest structures can be managed by applying diverse silvicultural methods (Aguilera & Benavidez 2005). However, planning of silvicultural interventions and timber appraisal requires more accurate prediction of tree merchantable volumes. Most of the research carried out to develop volume models have been focused on tree species belonging to the Roble-Raulí-Coigüe forest type - after the common name of its three major tree species, i.e. *Nothofagus obliqua* (Mirb.) Oerst, *Nothofagus nervosa* ((Phil.) Dim. et Mil.) and *Nothofagus dombeyi* (Mirb.) Oerst. (Kahler, 1993; Salas, 2000; Hueitra, 2004; Gezán et al., 2009). Despite the importance of the Roble-Hualo forest type in terms of the area covered and the potential growth rate, few studies have developed volume models. Most of these studies for Roble and Hualo tree species have focused on the development of volume equations for predicting total volume or volume until a given merchantable top diameter (Flández, 1999; Drake et al., 2003; Corvalán, 2015). In addition, few studies have reported the development of taper equations models for these tree species (Higuera, 1994; Vallejos et al., 2000). Most of the developed volume models are based on a small sample of data representative of a specific geographic area. However, forest resource management requires volume models that can be applied across the entire range of geographic distribution and site conditions. In addition, most of the volume models developed do not allow for volume disaggregation to assess the potential recovery of high-value solid wood.

Modern forest management uses taper equations to predict upper stem diameters and stem volume at product-level (Kozak, 2004). Due to their flexibility taper equations have been incorporated into forest inventory processing systems and growth simulation systems (Trincado & Burkhart, 2006). Another approach is the development of compatible volume and taper equation systems. The concept of compatibility was introduced in forestry by Clutter et al. (1983) and implies that the volume obtained from the integration of the taper function from the base to the total height must equal the volume obtained using a total volume function. This compatibility can be achieved by first estimating the parameters of a taper equation and then deriving a total volume function using integral calculus. This procedure ensures that the taper function and volume function are analytically consistent (Sharma & Oderwald, 2001). However, since the parameters of this system are obtained by minimizing the sum of errors of stem diameter and not volume, the volume prediction can be biased. To avoid this issue, diverse procedures for a simultaneous parameter estimate for the equations that conform the system have been proposed (Fang et al., 2000). This allows for the minimization of the sum of squared errors to be performed simultaneously for both stem diameters and volume. A compatible volume estimation system might consist of a taper equation and a total volume function (Diéguez-Aranda et al., 2006) or a taper equation and a function to predict volume between two stem heights (Coble & Hilpp, 2006). In both cases, the volume function must be analytically derived from the taper equation used. More complex procedures for developing compatible volume systems have also been proposed (Nunes et al., 2010). According to our knowledge, the construction and evaluation of this type of compatible models for Chilean native forest tree species has not been reported.

The aim of this research was the development of a compatible volume and taper equation system for the native tree species roble and hualo to be applied in all range of geographical distribution of second-growth forests belonging to the Roble-Hualo forest type. The primary objectives were to (1) derive a compatible volume system based on a segmented and a high-degree polynomial taper model, (2) account for error autocorrelation incorporating a continuous autoregressive error structure CAR(*x*), and (3) compare the stem diameter and total volume predictive capability of the developed compatible systems.

METHODS

Data. Data for the construction of the compatible volume and taper equation systems were obtained from destructive sampling carried out in stands belonging to the Roble-Hualo forest type located in the Maule and Ñuble regions. This type of native forest is located in the coastal mountain range between 32º 50'S and 36º 30'S and in the Andes Mountain range between 34°30'S and 36°50'S. The first source of data corresponds to stem analysis data from the project "Construction and validation of diametrical growth functions and compatible volume systems for roble and Hualo in the VI and VII regions" (Trincado, 2022). For this project, 13 sampling points were selected within the study area and at each point three circular plots of 400 m^2 were established. For each tree within the plot the following variables were recorded: tree species code, DBH (cm), total height (cm), height at crown base (m), dominance and stem quality considering attributes of health condition and stem shape. Within each plot, between 1 to 3 roble and hualo trees were selected for further measurements. Only unforked, without broken tops and healthy trees without anomalies along the stem were selected for stem profile measurements. Trees were felled and diameter outside bark (*D*) at breast height (1.3 m) and total tree height (*H*) were recorded again for each sample tree. Afterwards, the stump height and a sequence of diameters over bark (dob) and inside bark (ib) along the stem were also recorded. The sequence of measurements along the stem considered a first measurement at the bole base (i.e. at stump height), a measurement at the midpoint between the bole base and the *D*, a measurement at *D*, a next measurement at 2 m and the following measurements at 1m intervals until reaching a small-end diameter over bark of 5 cm. For this project, a total of 47 roble individuals and 74 hualo individuals were selected and measured. A second source of stem profile data was obtained from the project "Diagnosis and characterization of the current state and proposals for methodologies to evaluate the native forest of Sociedad Forestal Millalemu S.A. Stage II Proposals and evaluations of silvicultural interventions" (Nuñez et al., 1992). The procedure used for measuring stem profiles was similar the previous study and is described in detail by Higuera (1994). From this study a total of 37 roble individuals and 122 hualo individuals with stem profile data measurements were selected. A third source of stem analysis data was provided by the project "Density management diagrams for branch size control in northern high Andean dominated by roble forests in the Maule region" (Corvalán, 2015). In this study, within each plot established, roble individuals from the diameter class corresponding to the upper quartile were selected for stem profile measurements. The first stem diameter measurement over and inside bark was taken at the stem base, a measurement at *D*, a next measurement at 2.3 m and after that at 1m intervals until reaching a small-end diameter over bark of 10 cm. From this project an additional of 60 roble trees were selected. Thus, for our study, a total of 144 roble trees and 196 hualo trees were available for developing and validating the compatible volume system. The volume of each section was estimated using the Smalian formula and the last section, assuming the shape of a cone (Avery & Burkhart, 2001). The relative stem profiles for each tree species are shown in Figure 1.

Finally, the available stem analysis data were randomly divided to facilitate evaluation of stem diameter and total volume predictions. For each tree species, 70% of the trees were randomly selected for model fitting and 30% were used for model validation (Table 1).

Compatible volume and taper equation systems. Two compatible volume and taper equation systems were developed and evaluated for predicting stem diameter and total tree volume. The first compatible system was derived from the segmented taper equation proposed by Max and Burkhart (1976). This model has been shown to accurately estimate upper stem diameters for several planted tree species as loblolly pine (Figueiro-Filho et al., 1996), *Pinus palustris* Mill. (Jiang et al., 2010) and *Liriodendron tulipifera* (Jiang et al., 2005). The model consists of three second-degree polynomials describing the lower, middle, and upper part of the stem. These polynomials are grafted together at two join points (α_1 and α_2) with the restric-

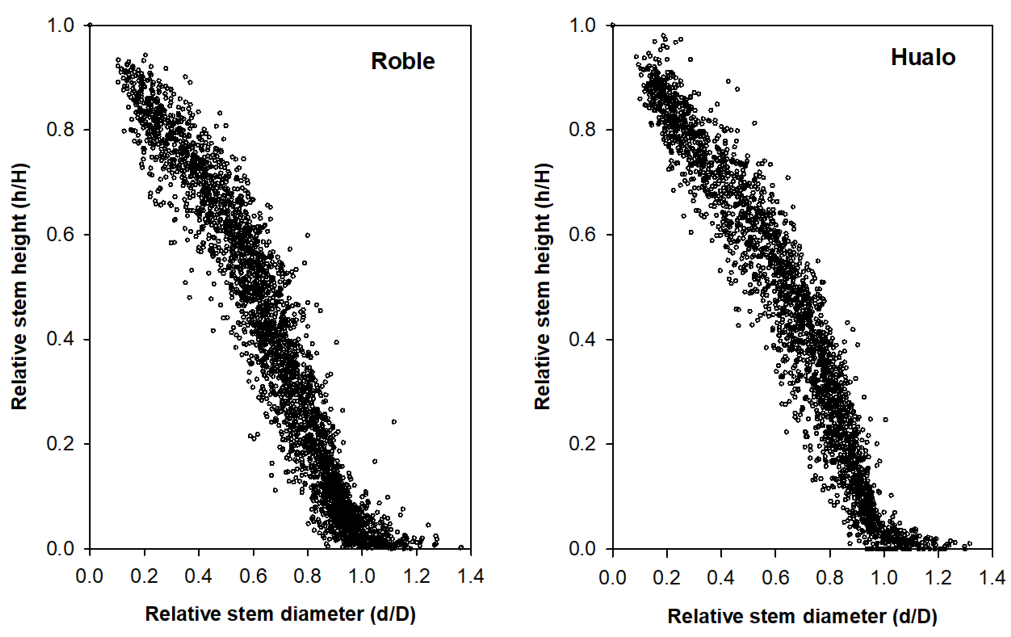

Figure 1. Relative stem profiles of sample trees for Roble (n = 144) and Hualo (n = 196).

Perfiles fustales relativos para los árboles muestra de Roble (n = 144) y Hualo (n = 196).

Table 1. Description of sample trees used for model fitting and validation.

Descripción de árboles muestras utilizados para el ajuste y validación de modelos.

Tree specie	Fitting			Validation		
	Mean ± SE	Min	Max	Mean ± SE	Min	Max
Roble	n = 101			n = 43		
D (cm)[a]	26.1 ± 0.7	9.8	48.5	24.2 ± 0.8	15.8	37.5
H (m)	17.6 ± 0.4	10.0	30.8	17.4 ± 0.5	11.3	24.5
Total inside bark volume (m^3)	0.43 ± 0.03	0.04	1.73	0.36 ± 0.03	0.08	0.78
Hualo	n = 137			n = 59		
D (cm)[a]	19.3 ± 0.6	6.3	42.7	20.0 ± 0.8	9.2	36.5
H (m)	14.5 ± 0.4	6.9	30.6	14.3 ± 0.6	6.5	30.3
Total inside bark volume (m^3)	0.21 ± 0.02	0.01	1.53	0.21 ± 0.03	0.03	1.09

[a] DBH measured over bark.

tions that the polynomials are continuous and have continuous first derivatives at each join point.

$$\frac{d^2}{D^2} = \beta_1(Z-1) + \beta_2(Z^2-1) + \beta_3(\alpha_1 - Z)^2 I_1 + \beta_4(\alpha_2 - Z)^2 I_2 + \varepsilon \quad [1]$$

I_i = 1 if (α_i-Z) ≥ 0, 0 otherwise (i = 1,2),

where D is the diameter at breast height outside bark (cm) measured at 1.3 m above the ground, d is the diameter inside bark (cm) at stem height h (m), H is the total tree height (m), $Z = h/H$, α_1, α_2, β_1..., β_4 are the parameters to be estimated and ε is the random error term. After performing the analytical integration of the taper equation, the following volume equation is derived

$$v(h_1,h_2) = K D^2 H \begin{bmatrix} \frac{\beta_1}{2}(Z_2^2 - Z_1^2) + \frac{\beta_2}{3}(Z_2^3 - Z_1^3) - (\beta_1+\beta_2)(Z_2 - Z_1) \\ -\frac{\beta_3}{3}\left\{(\alpha_1 - Z_2)^3 J_1 - (\alpha_1 - Z_1)^3 K_1\right\} \\ -\frac{\beta_4}{3}\left\{(\alpha_2 - Z_2)^3 J_2 - (\alpha_2 - Z_1)^3 K_2\right\} \end{bmatrix}, [2]$$

where $Z_1 = h_1/H$, $Z_2 = h_2/H$, h_1 is the lower stem height and h_2 is the upper stem height for determining the inside-bark volume (m^3) contained between to stem heights $v(h_1, h_2)$ considering $0 \leq h_1 \leq h_2 \leq H$ and $K = \pi/40.0000$. When $h_1 = 0$ y $h_2 = H$ the inside-bark total tree stem volume can be determined. The dichotomous variables contained in [2] must take the following values to assure for the correct use of the formula

$$J_i = \begin{cases} 1, & Z_2 \leq \alpha_i \\ 0, & Z_2 > \alpha_i \end{cases} \quad i=1,2$$

Using the same procedure, compatible volume systems have been developed for various pine and broadleaf tree species in Türkiye that showed a higher predictive capacity when compared to previously developed yield tables (Brooks et al., 2008; Özçelik & Brooks, 2012). A second compatible system was derived from the high-degree polynomial taper model proposed by Bruce et al. (1968). This model structure was selected because it has shown the best predictive capacity for other Chilean native tree species (Gezán et al., 2009). The high-degree polynomial taper equation has the following form

$$\frac{d^2}{D^2} = \beta_1 X^{1,5} + \beta_2(X^{1,5} - X^3)\cdot D + \beta_3(X^{1,5} - X^3)\cdot H + \beta_4(X^{1,5} - X^{32})\cdot H \cdot D + \beta_5(X^{1,5} - X^{32})\cdot H^{0,5} + \beta_6(X^{1,5} - X^{40})\cdot H^2 + \varepsilon, [3]$$

where $X = (H-h)/(H-1.3)$, β_1..., β_6 are the parameters to be estimated and ε is the random error term. After applying analytical integration of the taper equation, the following volume equation is derived

$$v(h_1,h_2) = -K(H-1.3)D^2 \begin{bmatrix} \beta_1 E_1 + (E_1 - E_2)\cdot(\beta_2 \cdot D + \beta_3 \cdot H) \\ +(E_1 - E_3)\cdot(\beta_4 \cdot H \cdot D + \beta_5 \cdot H^{0.5}) + \beta_6 \cdot H^2 (E_1 - E_3) \end{bmatrix} [4]$$

where the variables are defined as $E_1 = \frac{X_2^{2.5} - X_1^{2.5}}{2.5}$, $E_2 = \frac{X_2^4 - X_1^4}{4}$, $E_3 = \frac{X_2^{33} - X_1^{33}}{33}$, $E_4 = \frac{X_2^{41} - X_1^{41}}{41}$, $X_1 = h_1/H$ y $X_2 = h_2/H$ and all additional variables were defined previously.

Parameter estimate procedure. The developed system of equations has two components, a taper and volume equation that allows estimating volumes between two given stem heights ($0 \leq h_1 \leq h_2 \leq H$). The parameter estimation was performed by applying a simultaneous fitting procedure using the PROC MODEL included in SAS/ETS® statistical software (SAS Institute Inc. 2017). The fitting procedure used was seemingly unrelated regression (SUR) introduced in forestry by Borders (1989). This procedure was applied because the random errors of both functions are correlated, but none of the endogenous variables of one model appear as predictor variables in the other model (Zellner, 1962; Judge et al., 1988). In addition, the data used for developing the compatible systems correspond to multiple measurements along the stem that were taken from the same individual, which violates the assumption of independent observations assumed in regression analysis (Myers, 1990; Jayaraman & Zakrzewski, 1996). This causes the residuals of the taper model to be autocorrelated. Even when the estimated parameters are unbiased, their variance may be underestimated, which affects hypothesis testing and the construction of confidence intervals (Schabenberger & Pierce, 2001). The autocorrelation of the residuals was evaluated using the Durbin-Watson test and modeled by incorporating in the taper equations a continuous autoregressive error structure CAR(x). This structure assumes that the correlation of errors decreases with increasing distance between two consecutive measurements made on the same individual (Gregoire et al., 1995), and can be expressed as

$$e_{ij} = \sum_{k=1}^{x} I_k \rho_k^{h_{ij}-h_{ij-k}} e_{ij-k} + \varepsilon_{ij} \quad [5]$$

where e_{ij} is the j-th residual for the i-th tree (correspond to the difference between the observed and predicted stem diameter of the i-th tree at the j-th stem height), I_k = 1 for $j > k$ and 0 for $j = k$ (k = 1, 2, …, x), ρ_k is the autoregressive parameter of order k that must be estimated, h_{ij}-h_{ij-k} is the distance along the stem between the j-th height measurement and the j-th-k height measurement within each tree ($h_{ij} > h_{ij-k}$) and ε_{ij} random error (Pompa-García et al., 2012). For example, for a first-order autoregressive model CAR(x = 1) the above expression is defined as

$$e_{ij} = I_1 \rho_1^{h_{ij}-h_{ij-1}} e_{ij-1} + \varepsilon_{ij} \, ,$$

here ρ_1 is the first-order autoregressive parameter that must be estimated. Following the same procedure for a second-order autoregressive model CAR(x = 2) the error structure is defined as

$$e_{ij} = I_1 \rho_1^{h_{ij}-h_{ij-1}} e_{ij-1} + I_2 \rho_2^{h_{ij}-h_{ij-2}} e_{ij-2} + \varepsilon_{ij} \, ,$$

where I_2 = 1 for $j > 2$ and I_2 = 0 for $j \leq 2$, ρ_2 is the second-order autoregressive parameter that must be estimated, and h_{ij}-h_{ij-2} is the distance between the j-th and j-th-2 stem heights within the tree each i-th tree ($h_{ij} > h_{ij-2}$). In all cases, the error term ε_{ij} is considered a random variable normally distributed with expected mean equal to zero. According to Zimmerman & Núñez-Antón (2001) the selected error structure is appropriate for unbalanced and non-equidistant data which correspond to the type of data used to develop taper equations.

Evaluation of predictive capability. The predictive capability of the compatible volume and taper equation systems was evaluated using an independent data set of those used for parameter estimates (Table 1). The following statistics were used for the evaluation (Yang et al., 2009)

$$\bar{e} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right) \quad [6]$$

$$\bar{e}\% = 100 \cdot \frac{\bar{e}}{\bar{y}} \quad [7]$$

$$\text{RMSE} = \sqrt{\bar{e}^2 + \sigma_e^2} \quad [8]$$

where y_i is the observed value, $\hat{y}_i$ is the predicted value for the stem diameter or total stem volume for the i-the observation (i = 1,2,…,n), $\bar{y}$ is the observed sample mean for the stem diameter or total stem volume, $\bar{e}$ is the mean residual or mean error prediction, σ_e^2 is the variance of the error prediction. The mean residual and the sample variance can be considered estimates of bias and precision, respectively (Arabatzis & Burkhart, 1992). Finally, we calculated an estimate of the root mean square error (RMSE) combining measures of bias and precision. The RMSE was expressed as percentage of the sample mean and was the main criteria used for evaluation. For the developed taper equations, the stem diameter predictive capability was evaluated for relative height classes.

RESULTS

Parameter estimates for compatible systems. The parameter estimates for the taper and volume equations that make up the developed compatible systems, and the second-order autoregressive parameters CAR(2) are presented in Table 2. For both tree species, all parameters of the segmented taper equation proposed by Max and Burkhart (1976) were statistically significant (P ≤ 0.05). In contrast, some of the parameters of the compatible system derived from the high-degree polynomial proposed by Bruce et al. (1968) were statistically non-significant (P > 0.05). These results indicate a better goodness-of-fit of the segmented Max and Burkhart (1976) taper equation to the stem diameter data for both tree species. A detailed description of the goodness-of-fit statistics used here for model evaluation can be found in Myers (1990).

Table 2. Estimated parameters and goodness-of-fit statistics for the compatible systems including CAR(2).

Parámetros estimados y estadísticos de bondad de ajuste del sistema compatible incluyendo CAR(2).

Parameters	Roble		Hualo	
	Max & Burkhart (1976)	Bruce et al. (1968)	Max & Burkhart (1976)	Bruce et al. (1968)
β_1	-2.7474	0.866431	-2.89292	0.811306
β_2	1.272559	-0.00397	1.353921	0.009229
β_3	-1.35115	0.015969	-1.57318	-0.00239^{ns}
β_4	20.00919	-0.00002^{ns}	5.832213	-0.00004
β_5		0.009999		0.004014
β_6		-0.00012		-0.00006
α_1	0.649335		0.688040	
α_2	0.10317		0.206773	
ρ_1	0.384524	0.362367	0.260679	0.257672
ρ_2	0.392228	0.377153	0.249454	0.264354
Taper equation goodness-of-fit				
$S_{y.x}$	0.0629	0.0616	0.0926	0.0729
R^2	0.9528	0.9567	0.9018	0.9335
Volume model goodness-of-fit				
$S_{y.x}$	0.00541	0.00517	0.0035	0.0030
R^2	0.9502	0.9545	0.9557	0.9626

ns = non-significant at P < 0,05.

Modeling error autocorrelation. When the error autocorrelation was not considered during the process of regression parameter estimate the residuals were correlated as shown for the Max and Burkhart (1976) model fitted for roble in Figure 2 (left side). Contrarily, the incorporation of a second-order autoregressive structure CAR(2) allowed for reducing the error autocorrelation that occurs when diameter measurements are made on the same individual along the stem as shown in Figure 2 (right side). The same behavior was observed for both tree species and compatible systems developed. For both compatible systems, the incorporation of a second-order autoregressive error structure CAR(2) was enough to reduce the error autocorrelation. Another important effect observed was on the reduction of the parameter estimate standard errors (not shown).

Comparison of predictive capability. The comparison was made first with respect to the capability of the developed compatible systems to predict stem diameters at different relative heights. Measures of bias in absolute and in percentage values, and the root mean square error (RMSE) by relative height classes are presented in Table 3. As mentioned previously, the main criterion for comparison and evaluation purposes was the RMSE.

For roble the lowest values of RMSE were obtained for most of the relative heights using the segmented taper equation proposed by Max and Burkhart (1976). However, from a practical point of view these differences between the compatible systems can be considered minor. The basal stem diameters located below a relative height of 0.1 are underestimated by the segmented Max and Burkhart's taper equations and overestimated by the high-degree polynomial proposed by Bruce et al. (1968). Upper stem diameters from a relative height of 0,6 are overestimated by both compatible systems, increasing until the top. In general, the largest RMSE was observed in the middle section of the stem bole (0,5-0,8). The same comparative analysis is presented for hualo in Table 4.

The segmented Max and Burkhart (1976) taper equation developed for hualo presented for all relative height classes the lowest RMSE values. However, both models can be considered equally competitive in terms of predicting stem diameters. Prediction of basal stem diameters located below a relative height 0.1 presented for both compatible models the same behavior as roble. Stem diameters are slightly underestimated by the segmented Max and Burkhart's taper equations and overestimated by the high-degree polynomial proposed by Bruce et al. (1968). Contrarily, for both compatible systems upper stem diameters are not strongly overestimated like roble and an overestimate is observed from a relative height 0.8 up to the top. Similarly to roble, the largest RMSE was observed in the middle section of the stem bole (0.5-0.9).

A second comparison of the developed compatible systems was made in terms of their predictive capability of total stem inside-bark volume. For both tree species, both compatible systems showed similar values of bias and error (Table 5). The compatible system tended slightly to underestimate the stem total volume by 3.3 to 3.4% and 2.0 to 2.3% for roble and hualo, respectively. For roble, both compatible systems presented the same RMSE values, being therefore equally competitive. In contrast, for hualo the segmented Max and Burkhart (1976) taper equation presented a lower RMSE than the Bruce et al. (1968) taper equation.

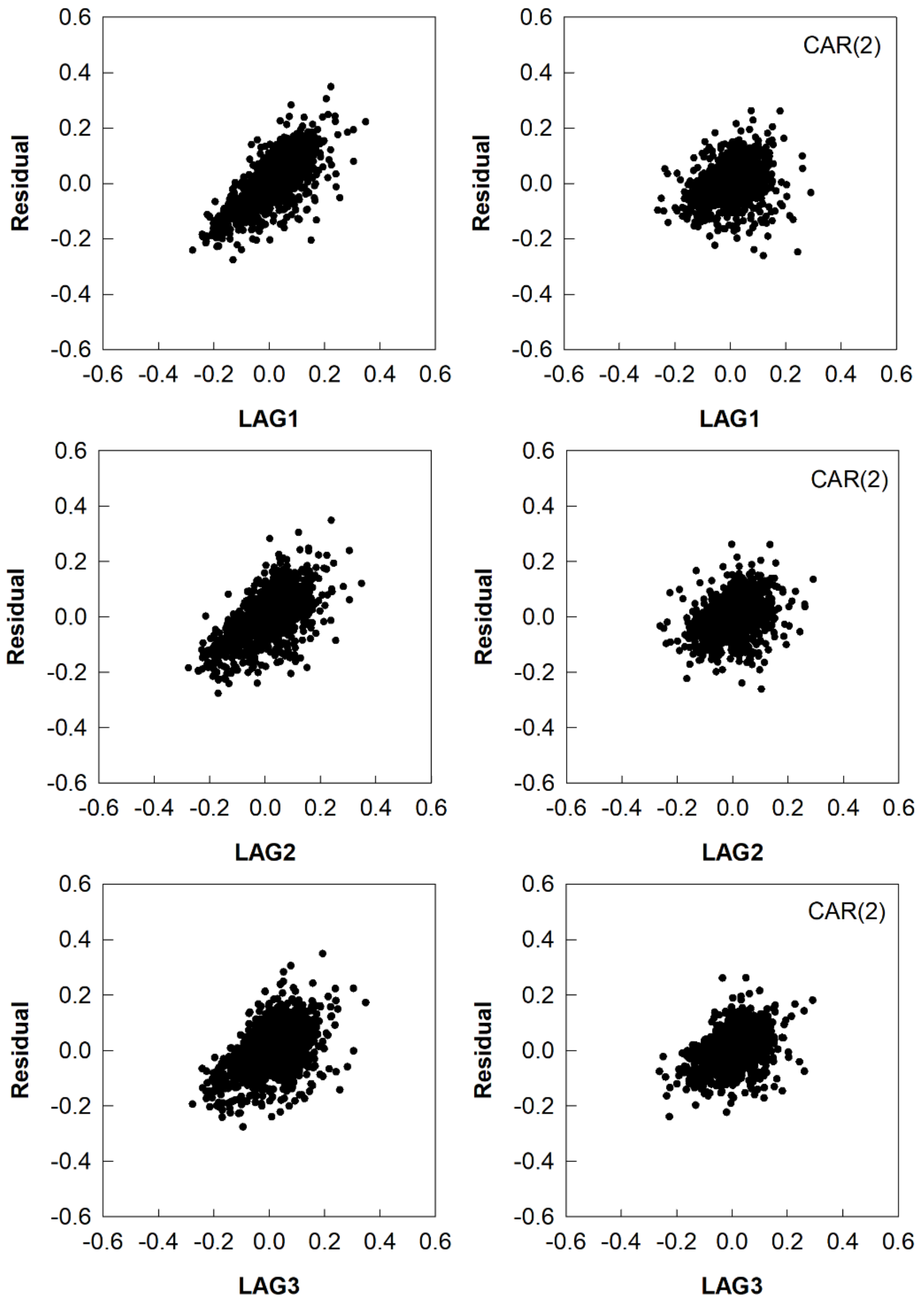

Figure 2. Autocorrelation errors for the Max & Burkhart (1976) taper model fitted for roble. On the left-side residuals for lags 1, 2 and 3, and on the right-side lags after incorporating CAR(2).

Autocorrelación de errores para el modelo de Max & Burkhart (1976) ajustado para roble. A la izquierda residuales con 1, 2 y 3 retardos y a la derecha con incorporación de CAR(2).

Table 3. Bias and error for inside-bark stem diameter predictions for roble by relative height classes. The lowest value of RMSE for each relative height class is highlighted in bold.

Sesgo y error para la predicción de diámetros fustales sin corteza para roble según clase de altura relativa. Se destaca en negrilla el menor valor de RMSE para cada clase de altura relativa.

Relative height	*n*	Max & Burkhart (1976)			Bruce et al. (1968)		
		$\bar{e}$ (cm)	$\bar{e}$ %	RMSE (cm)	$\bar{e}$ (cm)	$\bar{e}$ %	RMSE (cm)
$0.0 \leq h/H \leq 0.1$	108	0.53	2.22	**1.29**	-0.89	-3.72	2.03
$0.1 < h/H \leq 0.2$	75	0.16	0.72	**1.20**	0.31	1.45	1.21
$0.2 < h/H \leq 0.3$	73	0.,41	2.02	**1.23**	0.68	3.33	1.34
$0.3 < h/H \leq 0.4$	80	0.64	3.33	**1.62**	0.98	5.15	1.77
$0.4 < h/H \leq 0.5$	77	0.25	1.52	**1.99**	0.61	3.72	**1.99**
$0.5 < h/H \leq 0.6$	75	0.39	2.63	**2.13**	0.71	4.84	2.16
$0.6 < h/H \leq 0.7$	74	-0.06	-0.54	2.48	-0.01	-0.05	**2.43**
$0.7 < h/H \leq 0.8$	77	-0.23	-2.78	**1.96**	-0.49	-5.93	1.99
$0.8 < h/H \leq 0.9$	83	-0.88	-16.75	**1.42**	-1.16	-22.24	1.62
$0.9 < h/H < 1.0$	3	-0.60	-15.24	**1.57**	-0.75	-18.88	1.57
Total	725	0.14	0.88	**1.72**	0.03	0.16	1.87

Table 4. Bias and error for inside-bark stem diameter predictions for hualo by relative height classes. The lowest value of RMSE for each relative height class is highlighted in bold.

Sesgo y error para la predicción de diámetros fustales sin corteza para hualo clase de altura relativa. Se destaca en negrilla el menor valor de RMSE para cada clase de altura relativa.

Relative height	*n*	Max & Burkhart (1976)			Bruce et al. (1968)		
		$\bar{e}$ (cm)	$\bar{e}$ %	RMSE (cm)	$\bar{e}$ (cm)	$\bar{e}$ %	RMSE (cm)
$0.0 \leq h/H \leq 0.1$	194	0.48	2.34	**1.51**	-0.21	-1.04	1.73
$0.1 < h/H \leq 0.2$	95	0.21	1.17	**0.88**	-0.01	-0.09	0.95
$0.2 < h/H \leq 0.3$	88	0.28	1.72	**1.15**	-0.12	-0.72	1.18
$0.3 < h/H \leq 0.4$	88	0.07	0.46	**1.18**	-0.01	-0.10	1.20
$0.4 < h/H \leq 0.5$	94	-0.11	-0.80	**1.27**	0.05	0.40	1.40
$0.5 < h/H \leq 0.6$	87	0.19	1.62	**1.50**	0.48	4.07	1.72
$0.6 < h/H \leq 0.7$	83	0.06	0.59	**1.96**	0.21	2.23	2.13
$0.7 < h/H \leq 0.8$	80	0.12	1.57	**1.87**	0.02	0.28	1.99
$0.8 < h/H \leq 0.9$	68	-0.03	-0.48	**1.58**	-0.28	-4.96	1.73
$0.9 < h/H < 1.0$	10	-0.14	-3.04	**0.51**	-0.18	-3.85	0.52
Total	887	14.11	0.18	**1.28**	-0.01	-0.08	1.59

Table 5. Bias and error for total stem inside-bark volume predictions for roble and hualo. The lowest value of RMSE is highlighted in bold.

Sesgo y error para la predicción de volumen sin corteza para roble y hualo. Se destaca en negrilla el menor valor de RMSE.

Tree specie	Compatible system	Bias ($\bar{e}$)		Error (RMSE)	
		m^3	%	m^3	%
Roble (n = 43)	Max & Burkhart (1976)	0.012	3.3	**0.056**	**15.6**
	Bruce et al. (1968)	0.012	3.4	**0.056**	**15.6**
Hualo (n = 59)	Max & Burkhart (1976)	0.006	2.6	**0.026**	**12.4**
	Bruce et al. (1968)	0.004	2.0	0.036	17.1

DISCUSSION

This research developed new compatible volume and taper equation systems to be applied in second-growth forests of the Roble-Hualo forest type located in the Chilean central region. The compatible systems were derived from the segmented taper equation proposed by Max and Burkhart (1976) and from a high-degree polynomial taper equation proposed by Bruce et al. (1968). Parameter estimate was performed using the procedure *seemingly unrelated regression* (SUR). Until now, the development of compatible taper equation and volume systems has not been reported for native tree species in Chile and can be considered as a novel contribution of this research.

The modeling approach applied here accounted also for the error autocorrelation which originates from the multiple diameter measurements made along the stem on the same tree. The error autocorrelation was reduced considerably when including a second-order continuous autoregressive error structure CAR(2). This finding agrees with previous research work on taper equations which have incorporated successfully the same type of autoregressive error structures CAR(1) or CAR(2) for accounting for error autocorrelation (Garber & Maguire 2003; Rojo et al*.,* 2005). The development of taper equations for *Eucalyptus tereticornis* plantations in Colombia required the incorporation of a third-order continuous autoregressive error structure CAR(3) to eliminate the error autocorrelation (López et al., 2015). Accounting for error autocorrelation is important to meet among others the assumption of error independence in regression analysis. However, Kozak (1997) demonstrated that the predictive capability of a variable-exponent taper equation was not seriously affected when using autocorrelated data. On the other hand, William & Reich (1997) investigated the effect that the correlated error structure has on the fit of a taper equation. They found that accounting for the correlated errors structure in the equation fitting process significantly improved fit.

In this study, for both tree species some of the parameters of the high-degree polynomial taper equation proposed by Bruce et al. (1968) were statistically non-significant ($P > 0.05$). In contrast, all parameters for the segmented taper equation proposed by Max and Burkhart (1976) were statistically significant ($P < 0.05$). Demonstrating a better goodness-of-fit of this taper equation to the stem analysis data for both tree species. However, this issue should not limit its use for predicting stem diameters and stem volume. In general, the stem diameter and total stem volume predictive performance of this model was slightly lower in comparison to the Max and Burkhart (1976)'s segmented taper equation. Both compatible systems for roble overestimated stem diameters at relative heights equal to or greater than 0.8 showing a bias over 15%. However, this behavior was not observed for hualo, where the bias was lower than 5% for the same relative height. This finding can indicate stem shape differences between both native tree species that need to be considered in future studies. Additionally, for both tree species the largest errors were observed in the middle portion of the stem bole. It can be explained by the branching habits of deciduous tree species that might influence the stem shape. Regarding total stem volume prediction, the developed compatible systems presented for both tree species a similar performance in terms of bias and error (RMSE). For both compatible systems the bias was low and did not exceed 3.4% and 2.6% for roble and hualo, respectively. They showed a slight tendency to underestimate total stem volume and the error in percentage values was of 15.6% for both compatible systems for roble, and 12% for the compatible system derived from the segmented taper equation and 17% for the compatible system derived from the high-degree polynomial taper equation for hualo.

The comparison performed between the two developed compatible systems in terms of goodness-of-fit and goodness-of-prediction indicates that the compatible system derived from the segmented model proposed by Max and Burkhart (1976) was the best. For both tree species, the average predicted curve applying the Max and Burkhart's segmented taper equation overlapped to the validation data is shown in Figure 3.

In previous research, Gezán et al. (2009) determined that for the main tree species of the Roble-Raulí-Coihue forest type the Bruce et al. (1968) taper model and the variable-exponent taper model proposed by Kozak (1988) presented the best predictive capacity compared to other types of stem models. However, in their study the segmented model proposed by Max and Burkhart (1976) was not considered in the comparison.

At the beginning our research aimed to compare the predictive capacity of the developed compatible systems against previously reported taper equations. However, for these tree species, forest type and geographic location, the only studies conducted have been reported by Vallejos et al. (2000) and Higuera (1994). In the case of Vallejos et al. (2000), the taper equation developed modeled the stem diameters over bark, and not inside bark as was done in the present study. This makes it unfeasible to compare the predictive capacity with the compatible volume and taper equation systems developed here. In practice, the estimation of stem diameters and volumes without bark is required. On the other hand, it was not possible to compare them with the taper equations developed by Higuera (1994) because part of his data was used for fitting and validation in our study. Finally, it would be advisable to develop and evaluate new compatible volume and taper systems using other taper equation forms (Burkhart & Tomé 2012).

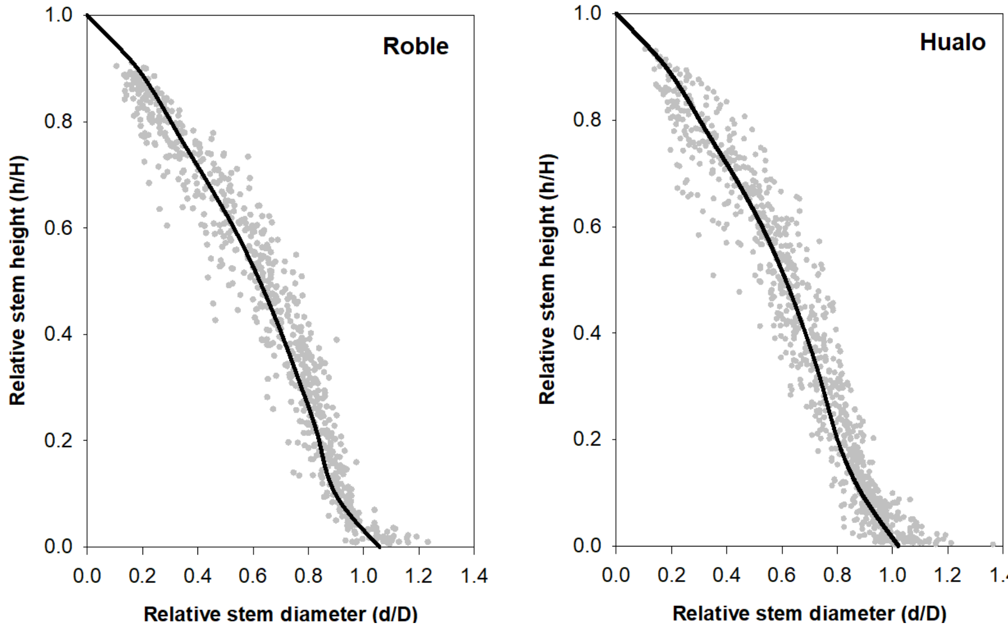

Figure 3. Predicted curve applying the Max & Burkhart (1976) taper model over the relative stem profiles of the validation data.

Curva predicha aplicando el modelo fustal de Max & Burkhart (1976) sobre los perfiles fustales relativos de los datos de validación.

CONCLUSIONS

The compatible system derived from the segmented taper model proposed by Max and Burkhart (1976) showed comparatively for both tree species a better fit and predictive performance of stem diameters and total volume than the compatible system derived for the polynomial taper model proposed by Bruce et al. (1968). The incorporation of a second-order continuous autoregressive error structure CAR(2) reduced considerably the residual autocorrelation to meet the assumption of error independence in regression analysis. We conclude that the compatible system derived from the segmented taper equation can be applied to the whole geographical range and site conditions where second-growth forests of Roble-Hualo are located. In addition, we recommend its incorporation into forest inventory processing systems for estimating tree standing volume at the product level.

AUTHOR'S CONTRIBUTIONS

GT designed the study, derived formulas, performed statistical analyses, and drafted the manuscript, CV performed data preparation and manuscript review, HG contributed to results analysis and manuscript review, and PC provided stem profile data and manuscript review.

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REFERENCES

Aguilera, M., & Benavides, G. (2005). *Recopilación de experiencias silvícolas en el bosque maulino*. Proyecto conservación y manejo sustentable del bosque nativo, CONAF (Corporación Nacional Forestal, CL).

Arabatzis, A.A. & Burkhart, H.E. (1992). An evaluation of sampling methods and model forms for estimating height-diameter relationships in loblolly pine plantations. *Forest Science, 38*(1), 192–198. https://doi.org/10.1093/forestscience/38.1.192

Avery, T.E., & Burkhart, H.E. (2001). *Forest measurements* (5th ed.). McGraw-Hill.

Borders, B.E. (1989). Systems of equations in forest stand modeling. *Forest Science 35*(2), 548-556. https://doi.org/10.1093/forestscience/35.2.548

Brooks, J.R., Jiang, L., & Özçelik, R. (2008). Compatible stem volume and taper equations for Brutian pine, Cedar Lebanon and Cilica fir in Turkey. *Forest Ecology and Management, 256*(1-2), 147-151. https://doi.org/10.1016/j.foreco.2008.04.018

Bruce, R., Curtis, L., & Vancoevering, C. (1968). Development of a system of taper and volume tables for red alder. *Forest Science, 14*(3), 339-350. https://doi.org/10.5962/bhl.title.87932

Burkhart, H.E, & Tomé, M. (2012). *Modeling forest trees and stands*. Springer.

Coble, D.W., & Hilpp, K. (2006). Compatible cubic-foot stem volume and upper-stem diameter equations for semi-intensive plantation grown loblolly pine trees in East Texas. *Southern Journal of Applied Forest, 30*(3), 132-141. https://doi.org/10.1093/sjaf/30.3.132

CONAF (Corporación Nacional Forestal, CL). (2021). *Catastro de los recursos vegetacionales nativos de Chile al año 2020*. Departamento de monitoreo de ecosistemas forestales.

Corvalán P. (2015). *Diagrama de manejo de la densidad de rodal para el control del tamaño de ramas basales en Bosques septentrionales altoandinos dominados por roble en la Región del Maule*. Serie Biomasa, Departamento de Gestión de Bosques y su Medio Ambiente, Facultad de Ciencias Forestales y de la Conservación de la Naturaleza, Universidad de Chile.

Clutter, J.L., Fortson, J.C., Pienaar, L.V., Brister, G.H., & Bailey, R.L. (1983). *Timber management: a quantitative approach*. Wiley.

Diéguez-Aranda, U., Castedo-Dorado, F., Álvarez-González, J.G., & Rojo, A. (2006). Compatible taper function for Scots pine plantations in northwestern Spain. *Canadian Journal of Forest Research*, *36*, 1190-1205. https://doi.org/10.1139/x06-008

Drake, F., Emannuelli, P., & Acuña, E. (2003). *Compendio de funciones dendrométricas del bosque nativo*. Universidad de Concepción - Proyecto Conservación y Manejo Sustentable del Bosque Nativo CONAF (Corporación Nacional Forestal, CL). KfW-DED-GTZ.

Fang, Z., Borders, B.E., & Bailey, R.L. (2000). Compatible volume-taper models for loblolly and slash pine based on a system with segmented-stem form factors. *Forest Science*, *46*(1), 1-12. https://doi.org/10.1093/forestscience/46.1.1

Figueiro-Filho, A., Borders, B.E., & Hitch, K.L. (1996). Taper equations for *Pinus taeda* plantations in Southern Brazil. *Forest Ecology and Management, 83*(1–2), 39–46. https://doi.org/10.1016/0378-1127(96)03706-1

Flández, A. (1999). *Evaluación de la exactitud para funciones de volumen de Nothofagus glauca (Phil.) Krasser y Nothofagus obliqua (Mirb.) Oerst, en la provincia de Linares* (Unpublished licentiate tesis). Universidad de Talca, Facultad de Ciencias Forestales, Escuela de Ingeniería Forestal.

Garber S.M., & Maguire, D.A. (2003). Modeling stem taper of three central Oregon species using nonlinear mixed effects models and autoregressive error structures. *Forest Ecology and Management, 179*, 507–522. https://doi.org/10.1016/S0378-1127(02)00528-5

Gezán, S.A., Moreno, P.C, & Ortega, A. (2009). Modelos fustales para renovales de roble, raulí y coigüe en Chile. *Bosque, 30*(2), 61-69. https://dx.doi.org/10.4067/S0717-92002009000200001

Gregoire, T.G., Schabenberger, O., & Barrett, J.P. (1995). Linear modeling of irregularly spaced, unbalanced, longitudinal data from permanent plot measurement. *Canadian Journal of Forest Research*, *25*, 137-156. https://doi.org/10.1139/x95-017

Higuera, C. (1994). *Funciones de volumen y ahusamiento para roble (Nothofagus obliqua) y hualo (Nothofagus glauca)* (Unpublished ingeniero forestal thesis). Facultad de Ciencias Forestales, Universidad Austral de Chile.

Hueitra, R. (2004). *Funciones de volumen y ahusamiento para roble (Nothofagus obliqua [Mirb.] Oerst.) en zonas de crecimiento en la IX y X región* (Unpublished ingeniero forestal tesis). Facultad de Ciencias Forestales, Universidad Austral de Chile.

Jayaraman, K., & Zakrzewski, W.T. (1996). *Regression with autocorrelated errors: Application in stem-profile models* (Forest Research Note No. 53). Ontario Forest Research Institute.

Jiang, L., Brooks, J.R., & Wang, J. (2005). Compatible taper and volume equations for yellow-poplar in West Virginia. *Forest Ecology and Management, 213*(1-3), 399-409. https://doi.org/10.1016/j.foreco.2005.04.006

Jiang, L., Brooks, J., John, R., & Clark, A. (2010). Compatible taper and volume equations for young longleaf pine plantations in southwest Georgia. In: J.A. Stanturf (Ed.), *Proceedings of the 14th biennial southern silvicultural research conference* (General Technical Reports SRS–121, p.p. 375-378). U.S. Department of Agriculture, Forest Service, Southern Research Station.

Judge, G.G., Griffiths, W.E., Hill, R.C, Lütkephol, H., & Lee, T.E. (1988). *Introduction to the theory and practice of econometrics*. John Wiley & Sons.

Kahler, C. (1993). Determinación de una función de ahusamiento para renovales de roble y raulí. *Ciencia e Investigación Forestal, 7*(1), 117-133. https://doi.org/10.52904/0718-4646.1993.183

Kozak, A. (1988). A variable-exponent taper equation. *Canadian Journal of Forest Research*, *18*, 1363-1368. https://doi.org/10.1139/x88-213

Kozak, A. (1997). Effects of multicollinearity and autocorrelation on the variable-exponent taper functions. *Canadian Journal of Forest Research*, *27*, 619-629. https://doi.org/10.1139/x97-011

Kozak, A. (2004). My last words on taper functions. *The Forestry Chronicle, 80*(4), 507-515. https://doi.org/10.5558/tfc80507-4

López, A.M., Barrios, A., & Trincado, G. (2015). Modelos de perfiles fustales con una estructura autorregresiva para plantaciones de Eucalyptus tereticornis en Colombia. *Madera y Bosques, 21*(2), 73-88. https://doi.org/10.21829/myb.2015.212446

Max, T.A., & Burkhart, H.E. (1976). Segmented polynomial regression applied to taper equations. *Forest Science, 22*(3), 283-289. https://doi.org/10.1093/forestscience/22.3.283

Myers, R.H. (1990). *Classical and modern regression with applications*. Duxbury Press.

Nunes, L., Tomé, J., & Tomé, M. (2010). A system for compatible prediction of total and merchantable volume allowing for different definitions of tree volume. *Canadian Journal of Forest Research*, *40*(4), 747-760. https://doi.org/10.1139/X10-030

Nuñez, P., Real, P., Grez, R., & Krause, G. (1992). *Diagnóstico y caracterización del estado actual y proposiciones de me-*

todológicas para evaluar el bosque nativo de Sociedad Forestal Millalemu S.A. Etapa II Proposiciones y evaluaciones de intervenciones silvícolas (Informe Convenio No 207). Facultad de Ciencias Forestales, Universidad Austral de Chile.

Özçelik, R., & Brooks, J.R. (2012). Compatible volume and taper models for economically important tree species of Turkey. *Annals of Forest Science, 69*, 105-118. https://doi.org/10.1007/s13595-011-0137-4

Pompa-García,M., Vega-Muñoz, J., Soto-Gutiérrez, R., Trincado, G., & Cruz-Cobos, F. (2012). Estimates of the bark thickness in bole profiles of Oak in northern Mexico. *Research Journal of Forestry, 6*(2), 32-40. https://doi.org/10.3923/rjf.2012.32.4

Rojo, A., Perales, X., Sánchez, F., Álvarez, J.G., & von Gadow, K. (2005). Stem taper functions for maritime pine (Pinus pinaster Ait.) in Galicia (northwestern Spain). *European Journal of Forest Research, 124*, 177-186. https://doi.org/10.1007/s10342-005-0066-6

Salas C. (2000). *Construcción de ecuaciones de volumen para las especies del Bosque adulto mixto y del renoval de roble (Nothofagus obliqua [Mirb.] Oerst.) del predio Rucamanque, IX región de la Araucanía* (unpublished ingeniero forestal thesis). Facultad de Ciencias Agropecuarias y Forestales, Universidad de la Frontera.

Schabenberger, O., & Pierce, F.J. (2001). *Contemporary statistical models for the plant and soil sciences*. CRC Press LLC.

Sharma, M., Oderwald, R.M. (2001). Dimensionally compatible volume and taper equations. *Canadian Journal of Forest Research*, *31*, 797-803. https://doi.org/10.1139/x01-005

SAS Institute Inc. (2017). SAS/ETS® 14.3 User's Guide.

Trincado, G. (2022). *Construcción y validación de funciones de crecimiento diametral y volúmenes de productos compatibles para Roble (Nothofagus obliqua) y Hualo (Nothofagus glauca) en la VI y VII regiones* (Informe Final Proyecto 004-2017, Fondo de Investigación de Bosque Nativo, CONAF).

Trincado, G., & Burkhart, H.E. (2006). A generalized approach for modeling and localizing stem profile curves. *Forest Science, 52*(6), 670-682. https://doi.org/10.1093/forestscience/52.6.670

Vallejos, O., Aracena, P., & Flández, A. (2000). *Funciones de ahusamiento para especies de Nothofagus en las provincias de Curicó, Talca y Linares. Modelos y métodos estadísticos aplicados a Bosques Naturales* (Reunión Internacional UACh/ FONDEF).

Ulloa, F., (1984). Alcances sobre la disponibilidad de bosque nativo comercial en Chile. *Actas XI Jornadas Forestales*, *4*, 65-70.

William, M.S., & Reich, R.M. (1997). Exploring the error structure of taper equations. *Forestry Science, 43*(3), 378-386. https://doi.org/10.1093/forestscience/43.3.378

Yang, Y., Huang, S., Trincado, G., & Meng, S.X. (2009). Nonlinear mixed-effects modeling of variable-exponent taper equations for lodgepole pine in Alberta, Canada. *European Journal of Forest Research*, *128*, 415-429. https://doi.org/10.1007/s10342-009-0286-2

Zellner, A. (1962). An efficient method of estimating seemingly unrelated regressions and test for aggregation bias. *Journal of American Statistical Association, 57*(298), 348-368. https://doi.org/10.2307/2281644

Zimmerman, D., & Núñez-Antón, V. (2001). Parametric modelling of growth curve data: An overview. *Test*, *10*(1), 1-73. https://doi.org/10.1007/BF02595823

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